Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2018

CS3BS03/IT3BS06 Discrete Mathematics

Programme: B.Tech. Branch/Specialisation: CSE/IT

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Let I be the set of integers, such that $f: I \longrightarrow I$ defined by $f(x) = 5x + 3$. Then this function is **1**

(a) One-one onto (b) One-one not onto

(c) Many one onto (d) None of these.

ii. If $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1,3)\}$ then relation R is called **1**

(a) Reflexive relation (b) Anti-symmetric relation

(c) Identity relation (d) Symmetric relation.

iii. Let $(P, \leq)$ be a partially ordered set. An element n in P is said to be a minimal element if for any $x \in P$ **1**

(a) $n \leq x \Rightarrow x = n$ (b) $x \leq n \Rightarrow x < n$

(c) $x \leq n \Rightarrow x = n$ (d) $x \leq n \Rightarrow x \neq n$.

iv. In Boolean algebra the value of boolean expression $(a+b).\ a'.\ b'$ is **1**

(a) 1 (b) $a.b$ (c) 0 (d) $a+b$.

v. Let A be a non empty set and * be a binary operation on A. Then algebraic structure $(A, *)$ is called a semigroup if it satifies _ _ _ _ laws. **1**

(a) Closure and Associative (b) Commutative and Associative

(c) Closure and Inverse (d) Identity and Commutative.

vi. Which one of the following statements is false? **1**

(a) Set $G = \{1, 2, 3, 4\}$ forms a Group under addition modulo 5.

(b) A group which has a generating set consisting of a single element is called a Cyclic group.

(c) The algebraic structure of the set of natural number $(N, +)$ is a Groupoid.

(d) The multiplicative group $G = \{1, -1, i, -i\}$ is a Torsion group.

vii. A graph in which all nodes are of equal degrees is known as **1**
(a) Multigraph (b) Regular graph
(c) Complete graph (d) Euler graph.

viii. The maximum number of edges in a simple connected graph with 12 vertices is **1**
(a) 60 (b) 62 (c) 64 (d) 66.

ix. For recurrence relation $a_{r+2} - 6a_{r+1} + 8a_r = 0$, the respective degree and order are: **1**
(a) 1 and 1 (b) 2 and 2 (c) 1 and 2 (d) 2 and 1.

x. If $4\,{}^{n}P_3 = 5\,{}^{n-1}P_3$ then **1**
(a) n = 25 (b) n = 10 (c) n = 15 (d) n = 20.

Q.2 i. Show that the relation **4**
$R = \{(a, b): a - b = even\ integer\ and\ a, b \in I\}$
in the set *I* of integers is an equivalence relation.

ii. (a) By the Pigeonhole Principle find the minimum number of students required in a class to be sure that at least 5 will receive the same grade if there are possible grades are A, B, C and D? **6**
(b) If A, B, C are three sets then prove A×(B∩C) = (A×B) ∩ (A×C)

OR iii. In a class of 100 students, 39 play Tennis, 58 play Cricket, 32 play Hockey, 10 play Cricket and Hockey, 11 play Hockey and Tennis, 13 play Tennis and Cricket. Using Principle of Inclusion and Exclusion find how many students play **6**
(a) All the 3 games,
(b) Just one game,
(c) Tennis and Cricket and not Hockey?

Q.3 i. Prove that in a distributed lattice $(L, \leq)$ if an element has a complement then this complement is unique. **4**

ii. Use the Quine-McCluskey method to simplify the sum of products expansion $xyz + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$ **6**

OR iii. Define partial order relation. In a Boolean algebra$(B, \wedge, \vee, ')$ if a relation $\leq$ is defined by $a \leq b \Rightarrow a \vee b = b\,, a \wedge b = a$ **6**
then prove that the relation ≤ is a partial order relation in B.

Q.4 i. Prove that the set of all positive rational numbers forms a group under the composition defined by $a*b = ab/2$. **4**

ii. Define Normal subgroup. Prove that the intersection of any two normal subgroup of a group is a normal subgroup. **6**

OR iii. If the system $(R, +, .)$ be a ring *R*, then prove that **6**
(a) $a.0 = 0.a = 0\ \forall\, a \in R$.
(b) $a.(-b) = (-a).b = -(a.b)\ \forall\, a, b \in R$
(c) $(-a).(-b) = a.b\ \forall\, a, b \in R$.

Q.5 i. Prove that a tree with *n* vertices has exactly *n-1* number of edges. **4**

ii. If $\Delta(G)$ is the maximum degree of any vertex of a graph *G*, then prove that $\chi(G) \leq 1 + \Delta(G)$. **6**

OR iii. Define- Circuit, Multigraph, Complete graph, Hamiltonian graph with suitable example and diagram. **6**

Q.6 i. (a) Determine the generating function of the given numeric function a_r where **4**

$$a_r = \begin{cases} 2^r & if\ r\ is\ even \\ -2^r & if\ r\ is\ odd. \end{cases}$$

(b) Prove that ${}^{n}C_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$.

ii. Solve the recurrence relation $a_r - 5a_{r-1} + 6a_{r-2} = 2 + r,\ r \geq 2$ with boundary conditions $a_0 = 1 = 1$ and $a_1 = 1$. **6**

OR iii. Solve $y_{h+1} - y_h = h$ with $y_0 = 1$ by using the method of generating function. **6**

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Discrete Mathematics

CS3BS03 / IT3BS06

Programme - B. Tech. Branch - CSE / IT

Qu. ①
i b) one-one not onto
ii a) Reflexive relation & b)
iii c) $x \leq n \Rightarrow x = n$
iv c) 0
v a) closure and associative
vi a) set $G = \{1, 2, 3, 4\}$ forms a group under addition modulo 5
vii b) Regular graph
viii d) $\frac{n(n-1)}{2} = \frac{12(12-1)}{2} = 66$
ix c) 1 and 2.
x c) $n = 15$

$A \subseteq A \times A$ then R is a relation

Qu. ② i). R will be an equivalence relation if it is Reflexive, symmetric and transitive. —①

Reflexive Let $a \in I$ then $a - a = 0$ (even int)

$\therefore (a, a) \in R, \quad \forall a \in I$

Hence R is a reflexive relation. (+1)

Symmetric Let $a, b \in I$

If $a - b$ = even integer then $b - a$ is also an even int.

$\therefore (a, b) \in R \Rightarrow (b, a) \in R$ is true.

Hence R is symmetric relation (+1)

Transitive Let $a, b, c \in I$

If $a - b$ = even int & $b - c$ = even int then

$(a-b) + (b-c) = a - c$ = even int

$\therefore (a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ is true

Hence R is transitive relation. (+1)

ii) (a). Let grades be the pigeonholes & no. of students be the pigeons. Then $m = 4$, $n = ?$

By pigeonhole principle $\left(\frac{n-1}{m}\right) + 1 = 5$ (+1)

$$\frac{n-1}{m} = 4$$

$$n = 4m + 1 = 4(4) + 1 = 17$$ (+1)

(b). Let $(a,b) \in A \times (B \cap C) \Leftrightarrow a \in A, b \in (B \cap C)$

$\Leftrightarrow a \in A, (b \in B \text{ and } b \in C)$

$\Leftrightarrow (a \in A, b \in B) \text{ and } (a \in A, b \in C)$

$\Leftrightarrow (a,b) \in (A \times B) \text{ and } (a,b) \in (A \times C)$

$\Leftrightarrow (a,b) \in (A \times B) \cap (A \times C)$

(+4)

Hence $A \times (B \cap C) = (A \times B) \cap (A \times C)$

OR

iii) Let S be the set of all students. Then

$|T| = 39$, $|C| = 58$, $|H| = 32$, $|C \cap H| = 10$, $|H \cap T| = 11$, $|T \cap C| = 13$.

$|S| = |T \cup C \cup H| = 100$

(a) All the three games i.e. $(T \cap C \cap H) = ?$

$\because |T \cup C \cup H| = |T| + |C| + |H| - |T \cap C| - |T \cap H| - |C \cap H| + |T \cap C \cap H|$

$100 = 39 + 58 + 32 - 13 - 11 - 10 + |T \cap C \cap H|$ (+2)

$|T \cap C \cap H| = 5$

(b). Just one game.

Who play only cricket $= |C \cap T' \cap H'| = |C - T - H|$

$= |C| - |C \cap T| - |C \cap H| + |C \cap T \cap H|$

$= 58 - 13 - 10 + 5 = 40$

Who play only Tennis $= |T \cap C' \cap H'| = |T - C - H|$

$= |T| - |T \cap C| - |T \cap H| + |T \cap C \cap H|$

$= 39 - 13 - 11 + 5 = 20$

Who play only Hockey $= |H \cap C' \cap T'| = |H - C - T|$

$= |H| - |H \cap C| - |H \cap T| + |H \cap C \cap T|$

$= 32 - 10 - 11 + 5 = 16$ (+2)

Total no. of students who just play one game $= 40 + 20 + 16 = 76$

3. (c) Number of students who play Tennis and Cricket but not Hockey $= |T \cap C| - |T \cap C \cap H|$

$= 13 - 5 = 8$ (+2)

Qu. (3) i). given that $(L, \leq)$ is a distributed lattice where $a \in L$ & $(b \& c)$ are its complement

$a \wedge b = 0$, $a \vee b = 1$, $a \wedge c = 0$, $a \vee c = 1$ (+1)

To show complement is unique we have to show they are equal

$$
\begin{aligned}
b &= b \wedge 1 \\
&= b \wedge (a \vee c) \\
&= (b \wedge a) \vee (b \wedge c) \\
&= (a \wedge b) \vee (b \wedge c) \\
&= 0 \vee (b \wedge c) \\
&= (a \wedge c) \vee (b \wedge c) \\
&= (a \vee b) \wedge c \\
&= 1 \wedge c \\
&= c
\end{aligned}
$$

(+2) (+1)

Hence proved.

ii)

	Binary	Minterm
xyz	111	7
$x\bar{y}z$	101	5
$\bar{x}yz$	011	3
$\bar{x}\bar{y}z$	001	1
$\bar{x}\bar{y}\bar{z}$	000	0

(+1)

Step I

Group	Minterm	Variable
0	0	000
1	1	001
2	3	011
	5	101
3	7	111

Step II

Group	Minterm Pair	Variable
0	0, 1	00X
1	1, 3	0X1
	1, 5	X01
2	3, 7	X11
	5, 7	1X1

n

Step III

group	Minterm Pair	Variable
0	1, 3, 5, 7	x x 1
	1, 5, 3, 7	x x 1
1	0, 1	0 0 x

(+3)

Step IV Prime implicant table

P.I.	0	1	3	5	7
1, 3, 5, 7		✓	✓	✓	✓
0, 1	✓	✓			

$F = z + \bar{x}\bar{y}$

(+2)

OR iii) Partial order relation -

A relation R on a set A is called partial order relation, if a) R is reflexive i.e aRa, $\forall a \in A$

b) R is antisymmetric i.e. $aRb, bRa \Rightarrow a=b$, $a,b \in A$

c) R is transitive i.e $aRb, bRc \Rightarrow aRc$, $a,b,c \in A$ (+1)

To prove relation $\leq$ is partial order relation following properties should be satisfied.

a) Reflexivity. Let $a \in B$, we have

$a \wedge a = a$ $a \vee a = a$

$\Rightarrow$ $a \leq a$ (+1)

Hence $\leq$ is reflexive.

b) Anti symmetry. Let $a, b \in B$, we have

$a \leq b, b \leq a \Rightarrow a \wedge b = a, b \wedge a = b$ | $a \vee b = b$, $b \vee a = a$

$\Rightarrow a \wedge b = b \wedge a$ | $a \vee b = b \vee a$

$\Rightarrow a = b$ | $b = a$

Hence $\leq$ is anti-symmetric (+2)

5. c) Transitivity let $a, b, c \in B$, we have

$a \le b, b \le c \Rightarrow a \wedge b = a, b \wedge c = b$

$a \wedge b = a$

$a \wedge (b \wedge c) = a$

$(a \wedge b) \wedge c = a$

$(a) \wedge c = a$

$\Rightarrow a \le c$

$a \le b, b \le c \Rightarrow a \vee b = b, b \vee c = c$

$b \vee c = c$

$(a \vee b) \vee c = c$

$a \vee (b \vee c) = c$

$a \vee (c) = c$

$\Rightarrow a \le c$

(+2)

Hence $\le$ is transitive

Now It is proved that the relation $\le$ is a partial order relation in B.

Qu. (4) i) Let Q_+ denote the set of all positive rational nu.

Closure property for $a, b \in Q_+$, $\left(\frac{ab}{2}\right)$ is also in Q_+ (+1)

Therefore Q_+ is closed.

Associative property Let $a, b, c \in Q_+$. Then

$(a * b) * c = \left(\frac{ab}{2}\right) * c = \left(\frac{ab}{2}\right)\frac{c}{2} = \frac{abc}{4}$

$a * (b * c) = a * \left(\frac{bc}{2}\right) = \frac{a}{2}\left(\frac{bc}{2}\right) = \frac{abc}{4}$

(+1)

Associativity satisfied

Existence of identity - e will be identity element if $e \in Q_+$

Now $e * a = a \Rightarrow \frac{ea}{2} = a \Rightarrow ea = 2a \Rightarrow ea - 2a = 0$

$\Rightarrow e = 2$, since $a \in Q_+ \Rightarrow a \neq 0$

Now $2 \in Q_+$.

$2 * a = \frac{2a}{2} = a = a * 2$. Identity satisfied (+1)

Existence of inverse -

Let b is to be the inverse of a

$b * a = e = 2 \Rightarrow \frac{ba}{2} = 2 \Rightarrow b = \frac{4}{a} \in Q_+$

Now $\left(\frac{4}{a}\right) * a = \frac{4a}{2a} = 2 = a * \left(\frac{4}{a}\right)$

Each element in Q_+ is inversible.

Hence given set forms a group (+1)

(a) ii). Normal subgroup. A subgroup H of a group G is said to be normal subgroup of G if for every $x \in G, h \in H$

$xhx^{-1} \in H$ or $xHx^{-1} \subseteq H$ — (+2)

Let H & K be any two normal subgroups. $x \in G, y \in H \cap K$

$\because x \in G, y \in H \Rightarrow xyx^{-1} \in H$

$x \in G, y \in K \Rightarrow xyx^{-1} \in K$ (+2)

Now $xyx^{-1} \in H$, $xyx^{-1} \in K \Rightarrow xyx^{-1} \in H \cap K$

$\because x \in G, y \in H \cap K \Rightarrow xyx^{-1} \in H \cap K$, $H \cap K$ is normal subgroup (+2)

OR iii) (a). $0+0=0$

$a.(0+0)=a.0$

$a.0+a.0=a.0$

$a.0+a.0=a.0+0$

$a.0=0$. Similar for 2nd part (+2)

b). $a+(-a)=0$

$[a+(-a)].b=0.b$

$a.b+(-a).b=0.b$

$a.b+(-a).b=0$

$(-a).b=-(a.b)$. Similar for 2nd part (+2)

c). $(-a).(-b) = c.(-b)$ put $-a=c$

$= -(c.b)$

$= -[(-a).b]$

$= -[-(a.b)] = a.b$ (+2)

Qu. 5 i). By mathematical induction

$n=1$ $e=0$

$n=2$ $e=1$ (+1)

$n=3$ $e=2$

$(n-1)$ Theorem is valid for all trees having less than n vertic[es]

Now $T-e = T_1+T_2$

T_1 has n_1 vertices

T_2 — n_2 — such that $n_1+n_2=n$ (+1)

No of edges in $T_1 = n_1-1$

$T_2 = n_2-1$

Total — $T_1+T_2 = n_1+n_2-2 \Rightarrow T-e = n-2$

$T = e+n-2 = 1+n-2 = n-1$

(+2)

Hence proved

ii). By mathematical induction

$n=1,2$ $\qquad \chi(G) \le 1+\Delta$

$1 \le 1+0$ & $2 \le 1+1$ (+2)

Theorem clearly follows

$n=n-1$ Induction hypothesis.

$\chi(G-v) \le 1+\Delta$

(+2)

$n=n$ For this add one vertex v to $G-v$

$\because$ degree of each vertex is atmost Δ then Δ colours are required and one different colour is assigned to vertex v. Hence $\chi(G) \le 1+\Delta$ (+2)

OR iii) Circuit. It is a closed walk in which initial & final vertices are same. Degree of each vertex is 2.  (+1.5)

Multigraph - A graph having some parallel edges.  (+1.5)

Complete graph - If each vertex is connected to everyother vertex.  (+1.5)

Hamiltonian graph - It contain a closed walk that traverses every vertex of the graph exactly once except the starting vertex at which the walk terminates  (+1.5)

Qu. (6). i). (a) $A(z) = a_0 z^0 + a_1 z^1 + a_2 z^2 + \cdots$

Put $r = 0,1,2,3 \cdots$ then $a_0 = 1, a_1 = -2, a_2 = 4, a_3 = -8, a_4 = 16, a_5 = -32$ (+1)

$A(z) = 1 - 2z + 4z^2 - 8z^3 + 16z^4 - 32z^5 + \cdots$

$= 1 - 2z + (2z)^2 - (2z)^3 + (2z)^4 - (2z)^5 + \cdots$ (+1)

$= (1+2z)^{-1} = \dfrac{1}{1+2z}$

(b). $^{n-1}C_r + {}^{n-1}C_{r-1} = \dfrac{(n-1)!}{r!\,(n-1-r)!} + \dfrac{(n-1)!}{(r-1)!\,(n-r)!}$ (+1)

$= \dfrac{\cancel{(n-1)!}}{\cancel{r!\,(n-1-r)!}} = \dfrac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\dfrac{1}{r} + \dfrac{1}{n-r}\right]$

$= \dfrac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\dfrac{n}{(n-r)\,r}\right]$

$= \dfrac{n!}{r!\,(n-r)!} = {}^nC_r$ (+1)

8. ii) Homogeneous solution —

$m^2 - 5m + 6 = 0 \Rightarrow m = 2, 3$

$a_r^{(h)} = c_1 2^r + c_2 3^r$ (+2)

Particular solution — Trial solution $A_0 + A_1 r$ (Put it in given eq.) $2A_0 - 7A_1 + 2A_1 r = 2 + r$

$A_0 = \frac{11}{4}, \quad A_1 = \frac{1}{2}$

$a_r^{(P)} = \frac{11}{4} + \frac{1}{2} r$ (+2)

Total solution - $a_r = c_1 2^r + c_2 3^r + \frac{11}{4} + \frac{1}{2} r$ — ①

Put boundary conditions $c_1 + c_2 = -\frac{7}{4}$

$2c_1 + 3c_2 = -9/4$

Hence $c_1 = -3, \ c_2 = \frac{5}{4}$ (+2)

Now eq ① $a_r = -3\,2^r + \frac{5}{4} 3^r + \frac{11}{4} + \frac{1}{2} r$

OR iii) $Y(z) = \sum_{n=0}^{\infty} y_n z^n = y_0 z^0 + y_1 z^1 + y_2 z^2 + \cdots$ (+2)

$\sum_{n=0}^{\infty} y_{n+1} z^n - \sum_{n=0}^{\infty} y_n z^n = \sum_{n=0}^{\infty} n z^n$

$(y_1 + y_2 z + y_3 z^2 + \cdots) - Y(z) = z + 2z^2 + 3z^3 + \cdots$

$\frac{Y(t) - y_0}{z} - Y(t) = t(1 + 2t + 3t^2)$

$Y(t) - 1 - tY(t) = t^2 (1-t)^{-2}$ $[\because y_0 = 1]$

$(1-t) Y(t) = 1 + t^2 (1-t)^{-2}$

$Y(t) = \frac{1}{1-t} + \frac{t^2}{(1-t)^3} = (1-t)^{-1} + t^2 (1-t)^{-3}$ (+2)

$\sum_{n=0}^{\infty} y_n t^n = \sum_{n=0}^{\infty} t^n + t^2 \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} t^n$ (+1)

$= \sum_{n=0}^{\infty} t^n + \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} t^{n+2}$

equating coefficient of t^n

$y_n = 1 + \frac{1}{2}(n-1)n.$